ICA Commission on Education and Training

Basic Problem of Map Projections Teaching

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1. Introduction

2. What a map projection is?

3. What are map projections for?

4. What should be changed in map projections teaching ?

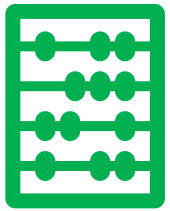
5. An example for the secondary school level

6. Conclusion

Proceedings of the Oklahoma Academy of Sciences many years ago (Freile 1954):

The mathematical ability of a geographer should include:

(a) Algebra
(b) Geometry
    Theoretical
        Plane
        Spherical
    Applied
        Solid
        Stereometry
(c) Curve Analysis and Determinants
(d) Trigonometry
    Plane
    Spherical
(e) Analytical Geometry
(f) Infinitesimal Arithmetic
(g) Probabilities



The parts of geography that need a mathematical background to insure an orderly comprehension are (Freile 1954):

(a) Form and shape of the Earth

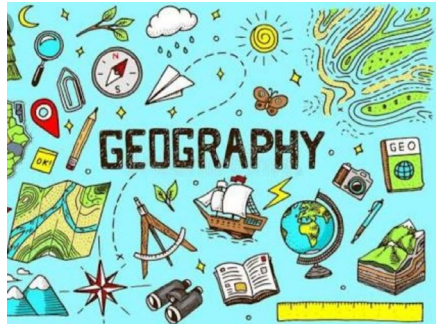
(b) Movements of the Earth and its immediate gravitational and electromagnetic relations

(c) Elements of longitude and variables of time determination

(d) Cartography and map interpretation

(e) Climatology

(f) Physical Geography



I am not sure if geographers of today are aware of stated

Opinions

- To keep the number of students, we need to avoid formulas and equations?
- If geography teachers are not well versed in mathematics, they will avoid interpreting to their students what they themselves are not strong enough at
- Aversion to change
- Proactive interference
- Lack of critical thinking
- etc

The basic problem of teaching map projections is the non-correlation with education in mathematics

2. What a map projection is?

The process of transferring geospatial data from a three-dimensional model of the Earth to a two-dimensional or “flat” map (Kraak et al, 2021)

But map projections are not limited to geospatial data and to the Earth. What about maps of the Moon or Mars? Why is "flat" in quotes?

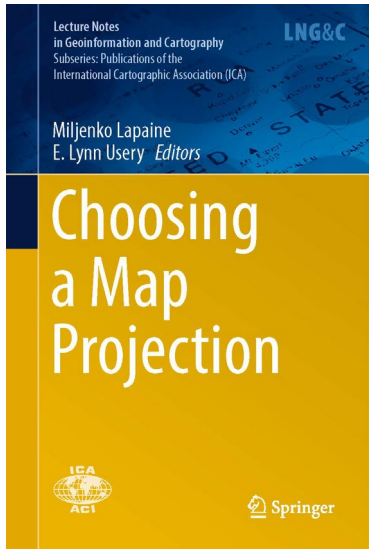
Mapping a curved surface, especially an ellipsoid or sphere, into a plane (Frančula et al, 2020)

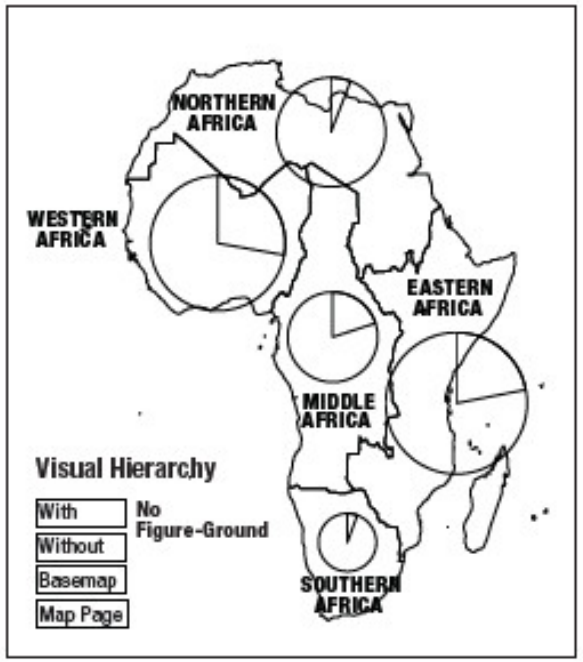
3. What are map projections for?

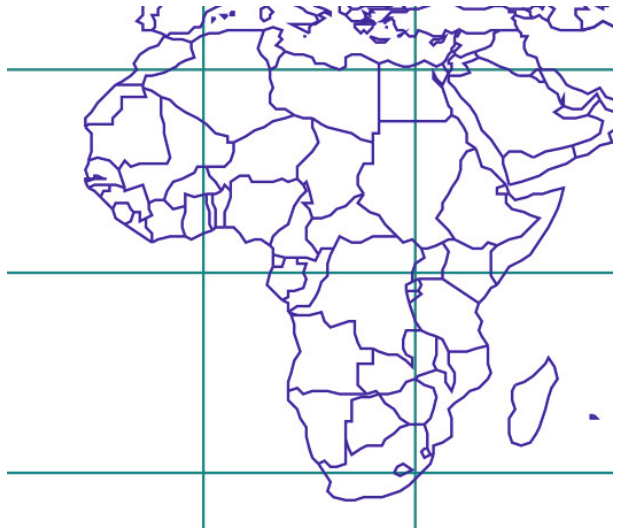
Map projections are used in making and analyzing maps

Choosing a projection from among the many options requires something like a cost-benefit analysis

Projections should be chosen to best serve the objectives, and just as important, to keep from making a serious error by selecting one that is inappropriate (Lapaine, Usery 2017)







Africa is approximately equally long in the east-west and in the north-south direction. The authors of the map on the left-hand side neglected the general rules on the choice of map projections.

4. What should be changed in map projections teaching?

Is it possible to teach cartography without using maps?

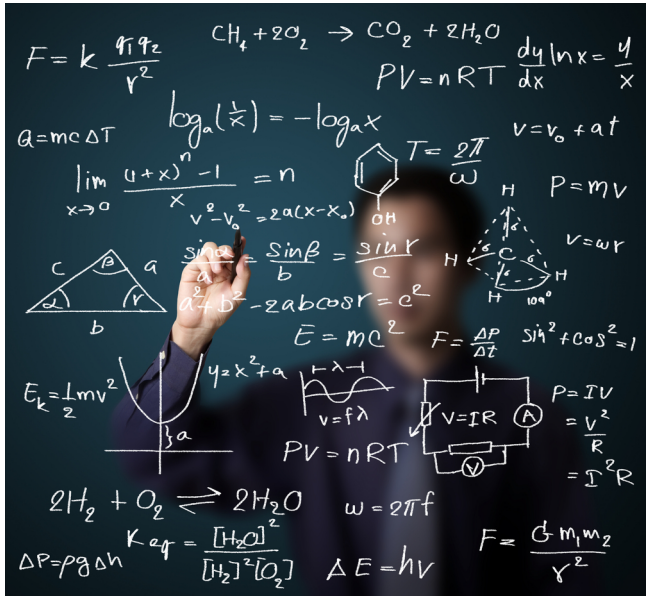
Is it possible to teach map projections without using mathematics?

(i) Explaining map projections without mathematics should be avoided.

If we interpret map projections without mathematics, then:

a) we underestimate those we teach and

b) we conceal our own lack of mathematical knowledge



(ii) The word *conceptually* should be left out, because it is unnecessary.

In map projections everything is real, there is no need for blurring.

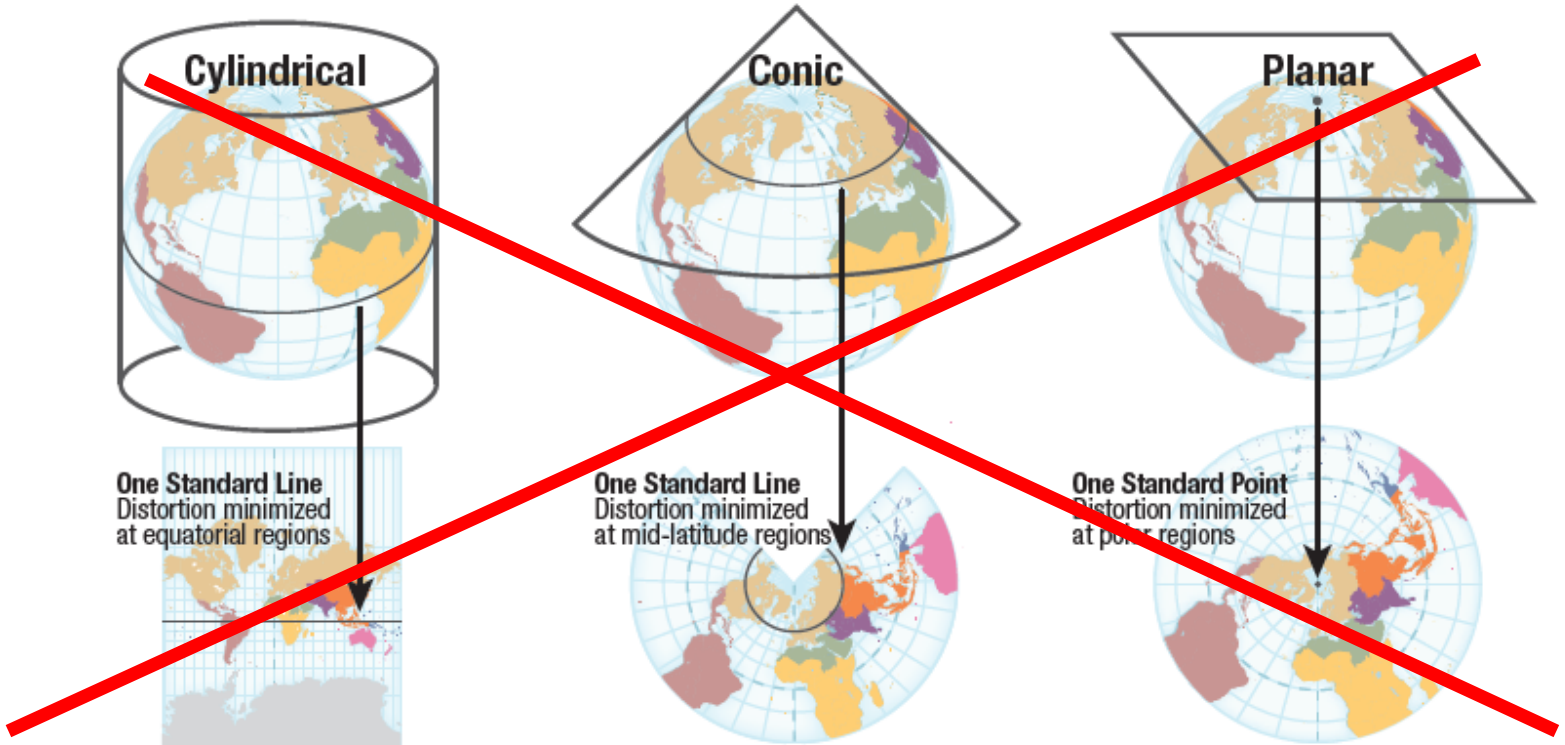
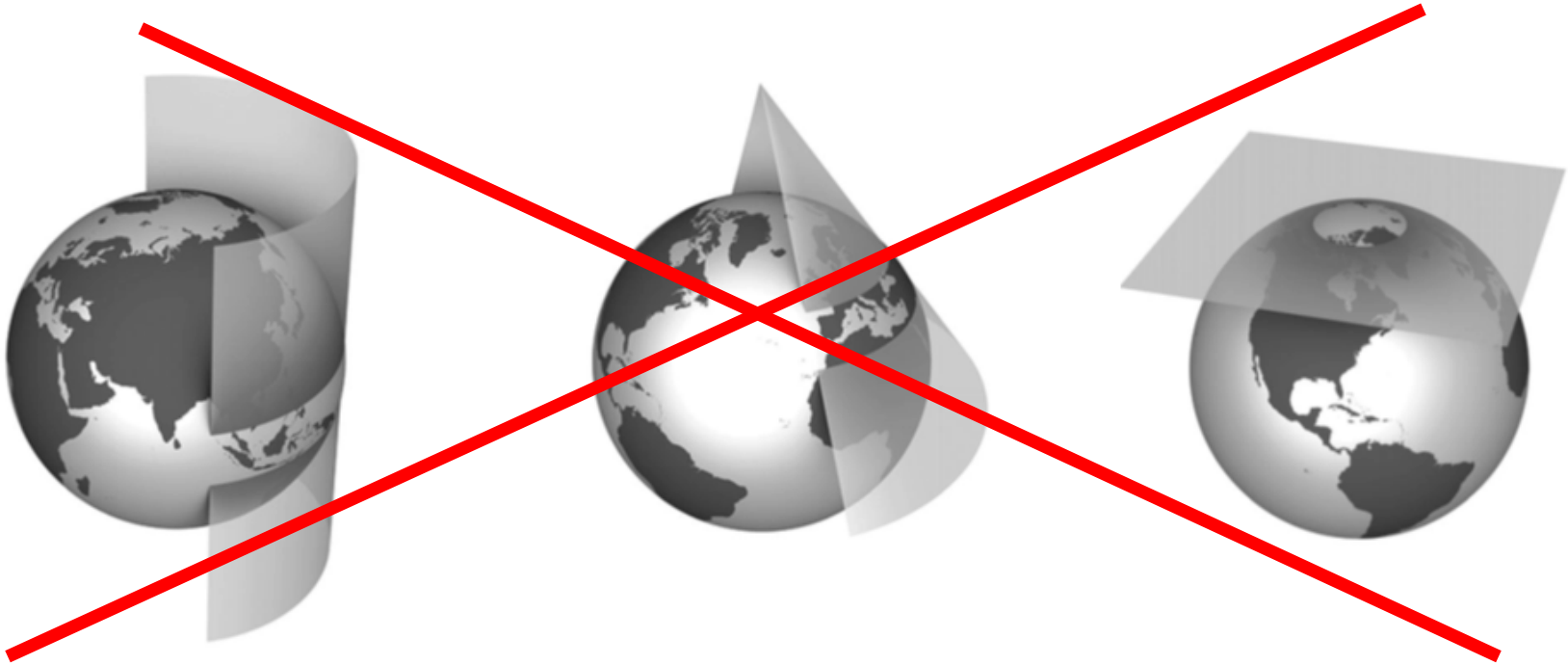
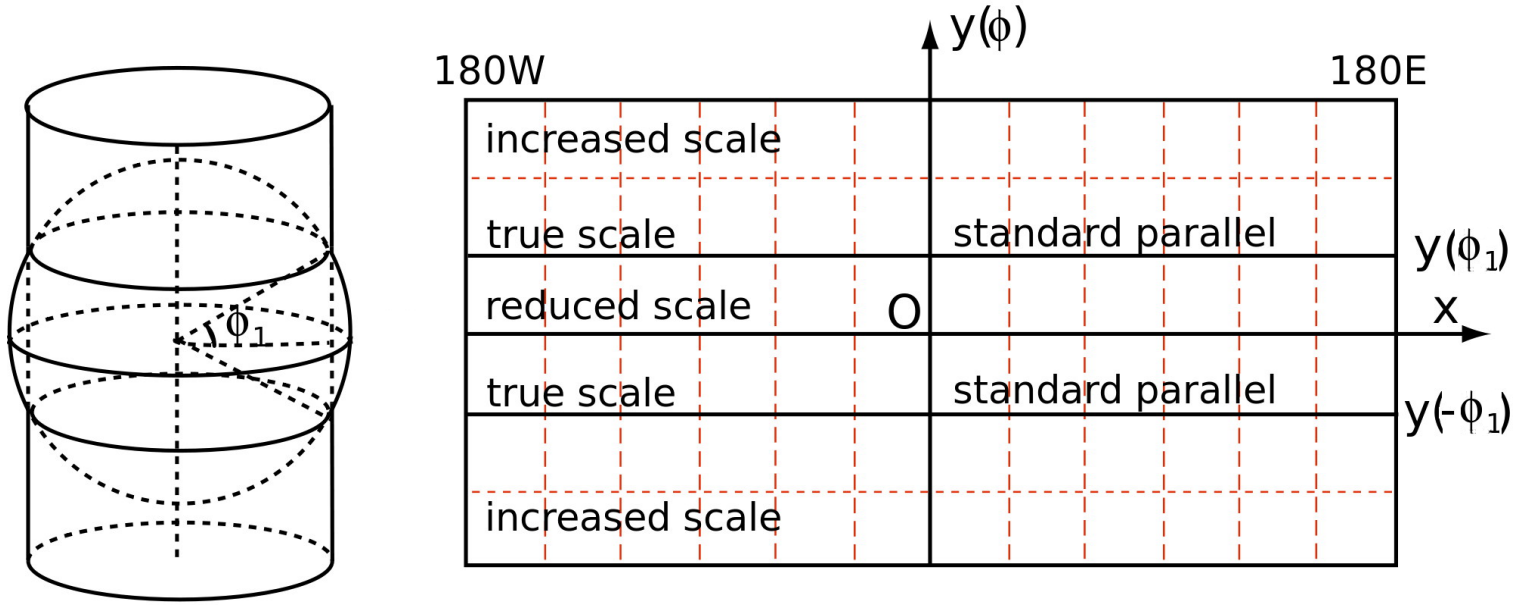


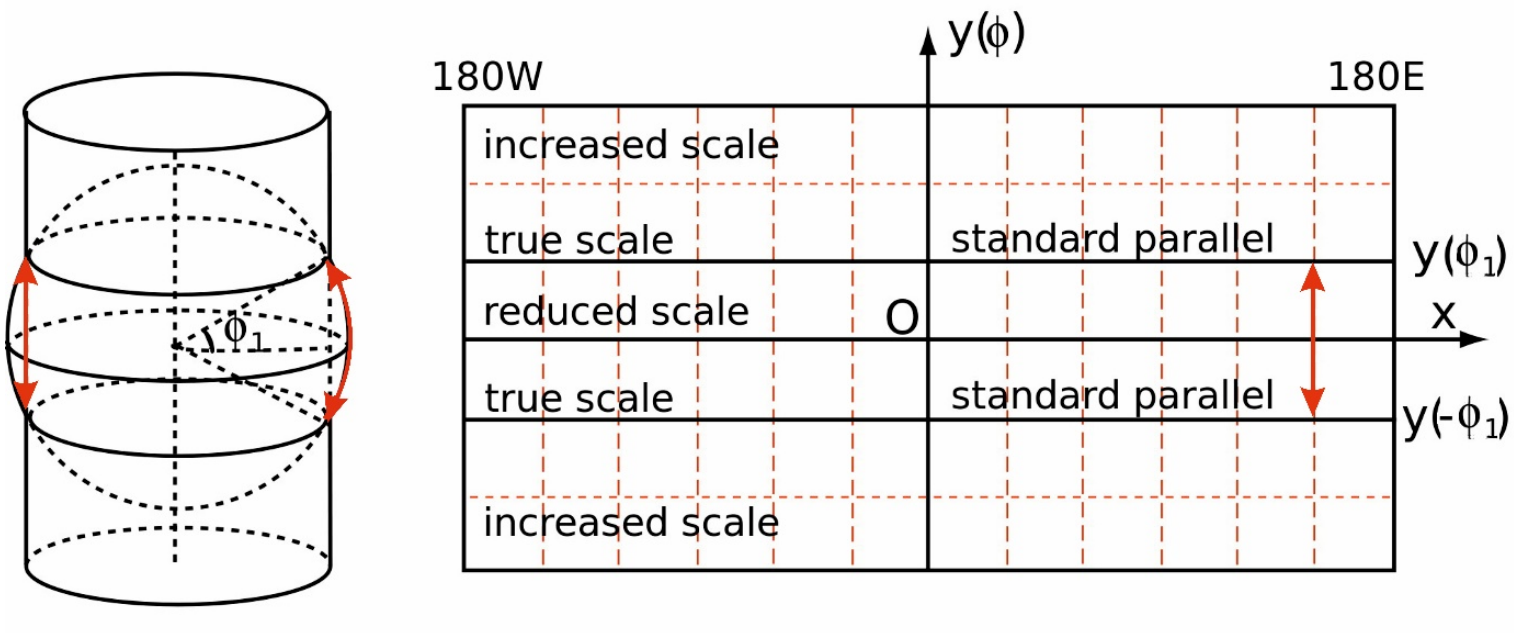
Figure 2.4-1. From globe to map surface. **Left:** Cylindrical (Mercator). **Centre:** Conic. **Right:** Planar. The graticule and standard lines or points are marked for comparison.



In general, there is no map surface in map projections except the plane. Less than 1% of all map projections are projections on the auxiliary or intermediate surface



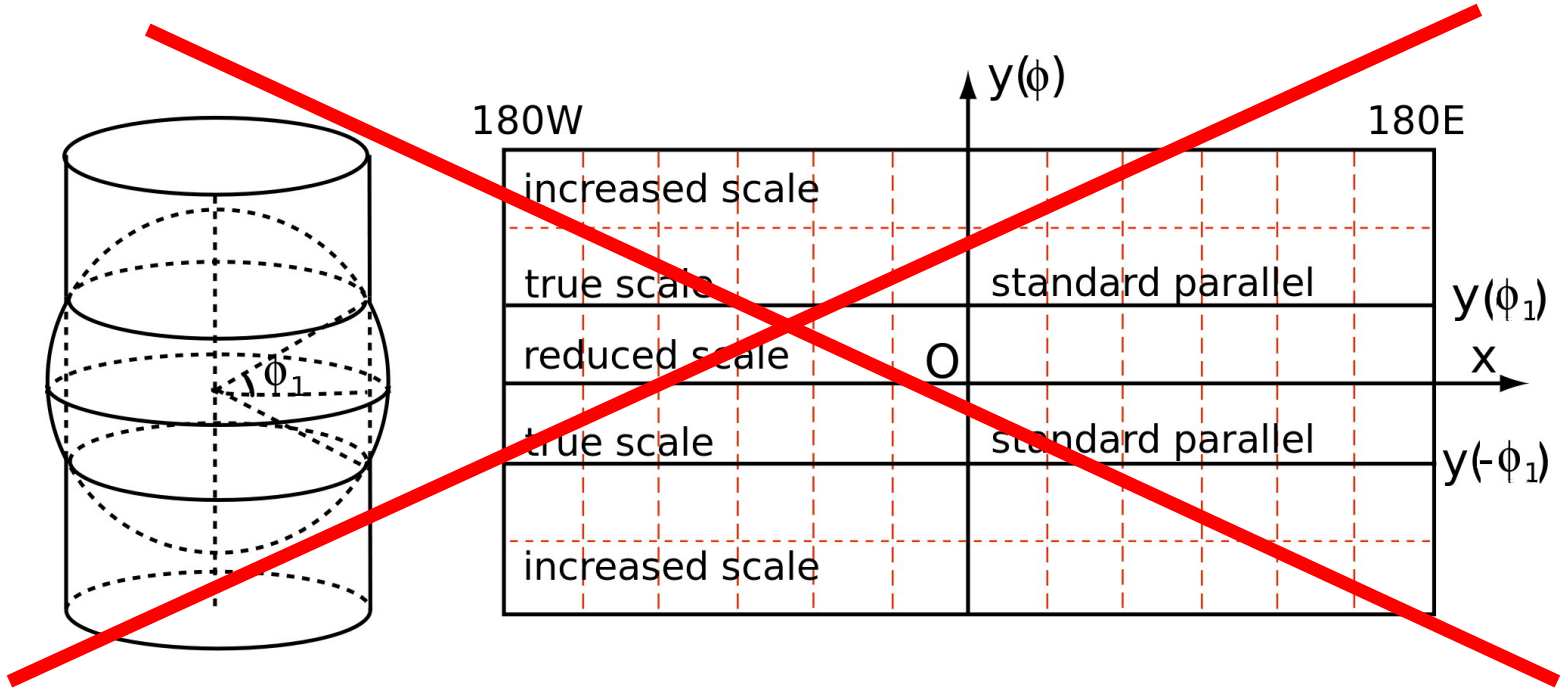
Optical illusion: standard parallels coincide with secant parallels



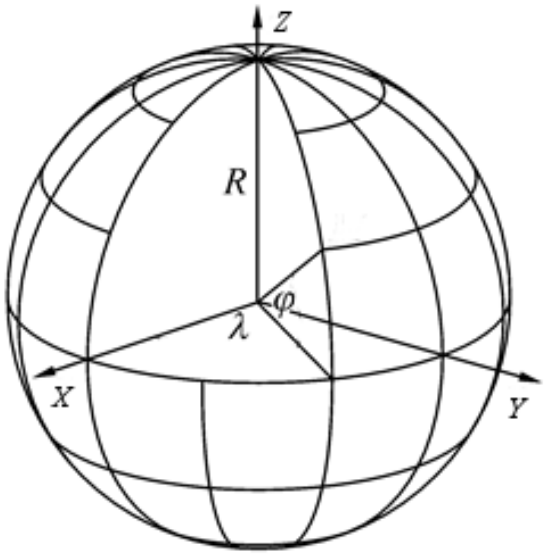
The distance between standard parallels are equal to the length of the circular arc?

(iii) Secant projection: a projection that slices the globe with the map surface, producing two standard lines for conic and cylindrical projections and one standard line for planar projections.

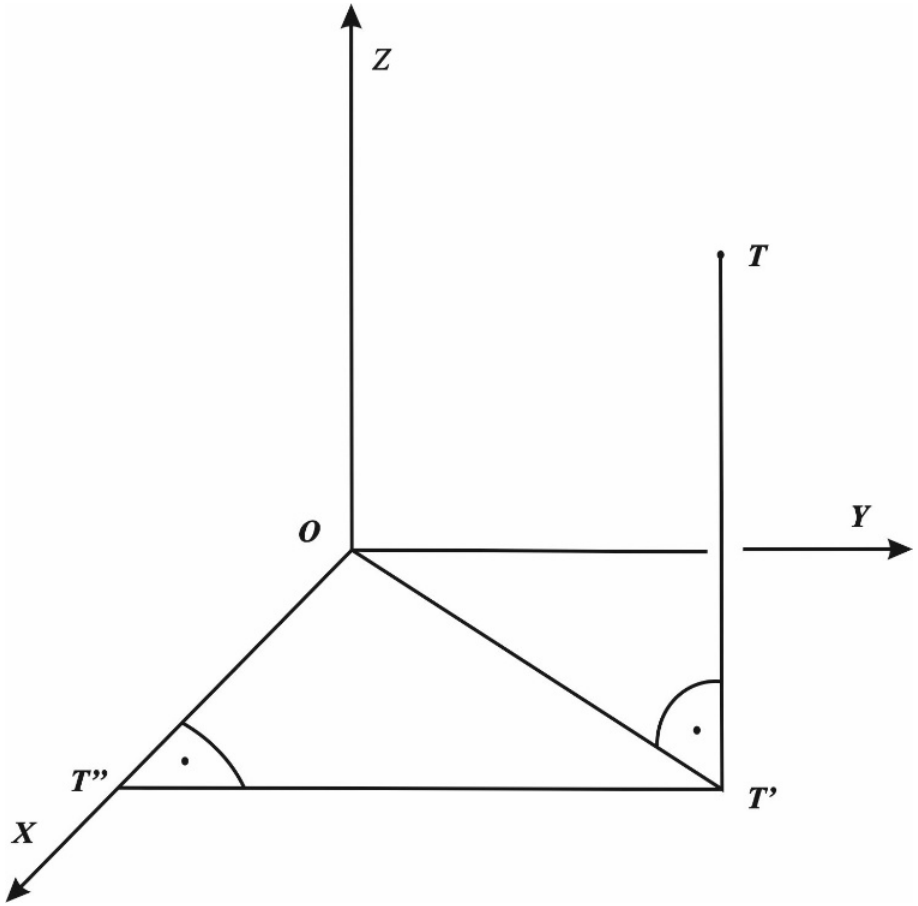
Such definition does not make sense, because secant line and standard line are two different concepts.



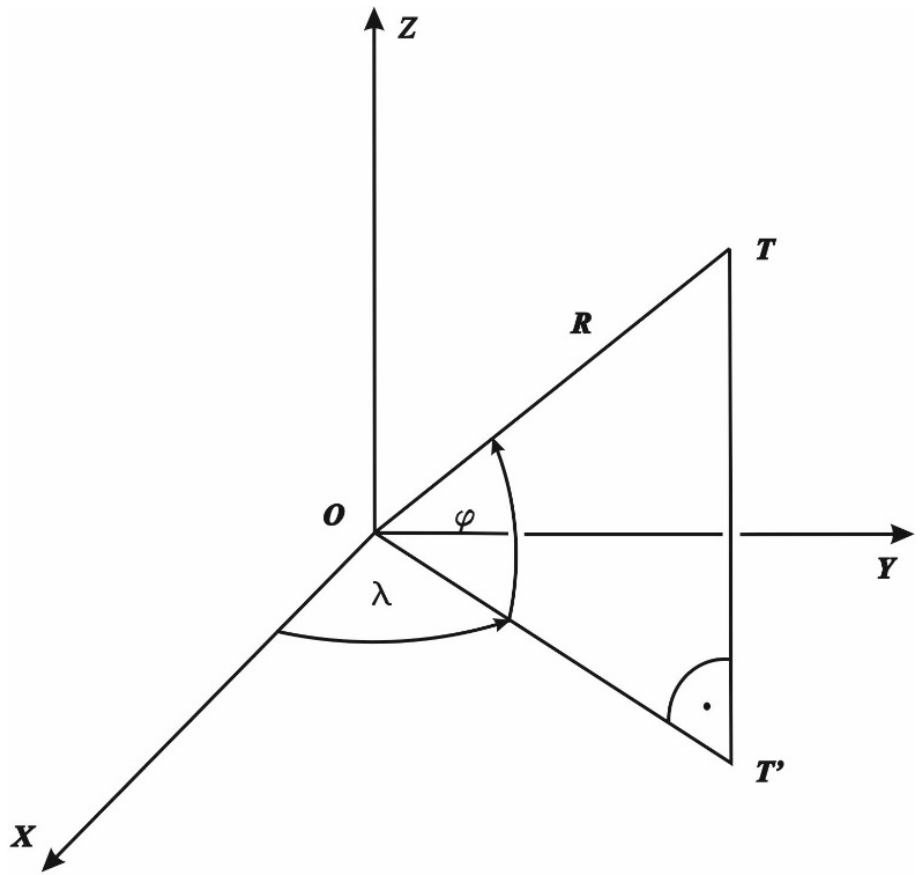
5. Proposal for the introduction of map projections in secondary schools



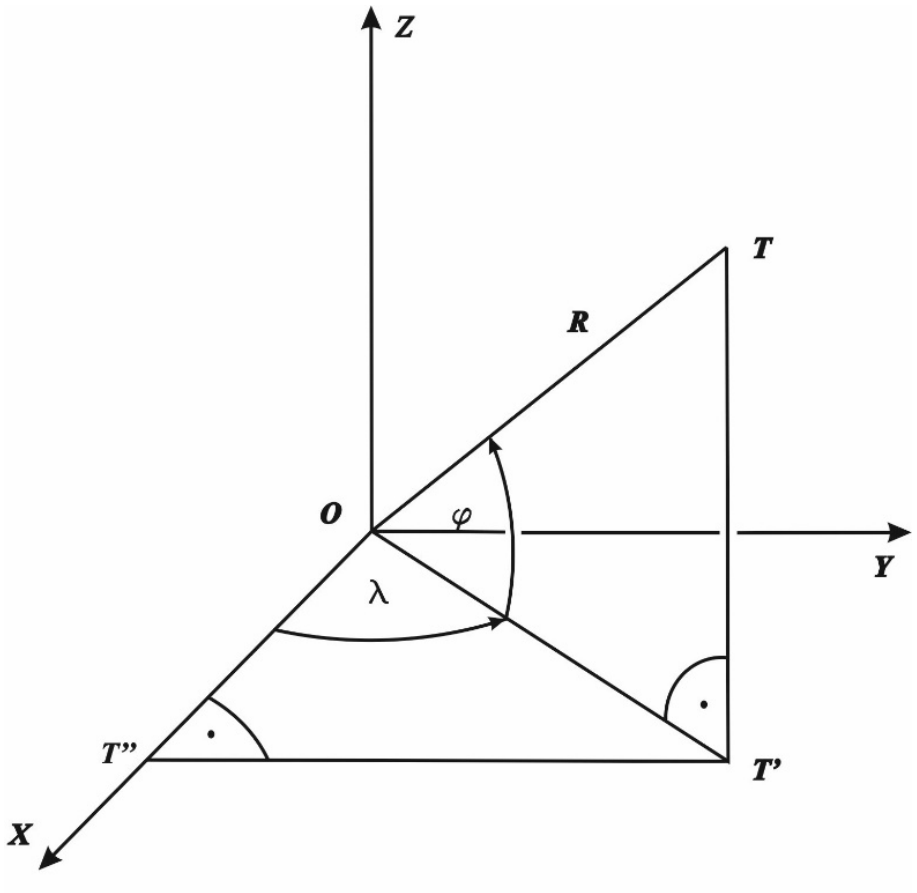
Elementary school intuitive approach: network of parallels and meridians, latitude ϕ and longitude λ. *R* is the radius of the sphere



The point *T* in a rectangular spatial coordinate system in mathematics is usually determined by the coordinates *X=OT''*, *Y=T''T'*, *Z=T'T*.



The point *T* in a spherical coordinate system in geography, geodesy, cartography is usually determined by coordinates *R*, λ and ϕ (*R*=const.>0, $\lambda \in [-\pi, \pi]$, $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$)



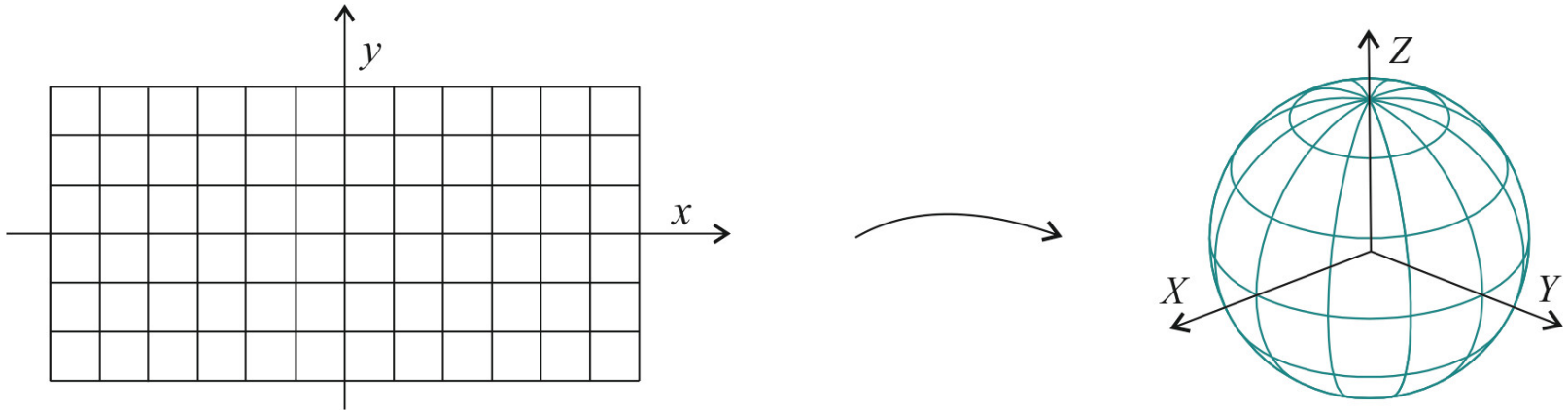
$$OT' = R\cos\varphi$$

$$X = OT'' = OT'\cos\lambda = R\cos\varphi\cos\lambda$$

$$Y = T''T' = OT'\sin\lambda = R\cos\varphi\sin\lambda \qquad (1)$$

$$Z = T'T = R\sin\varphi$$

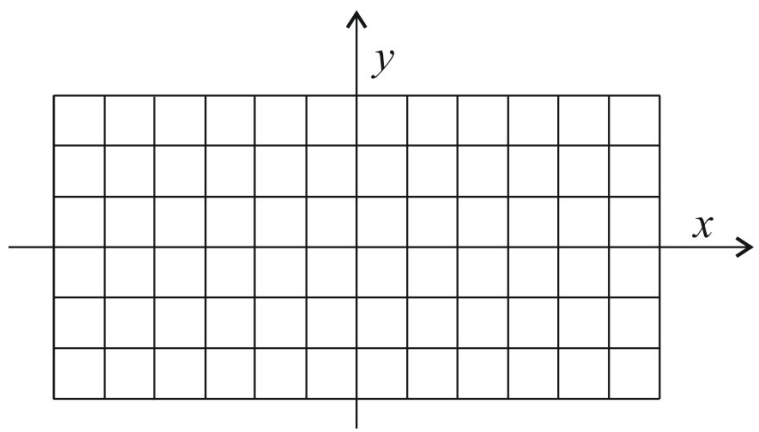
5.1. Geographic parameterization of a sphere

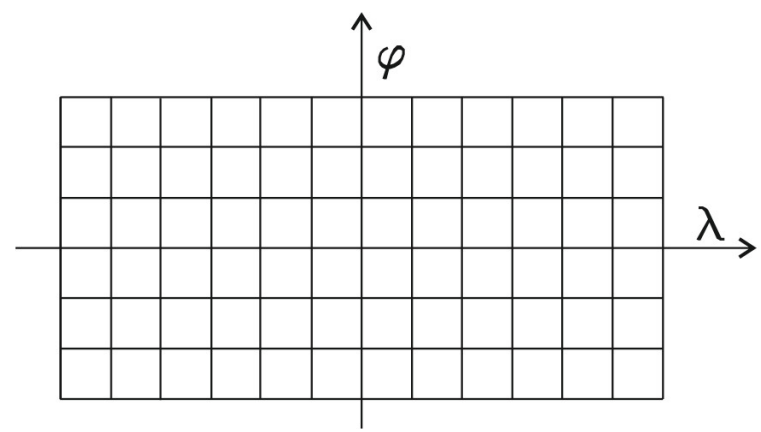


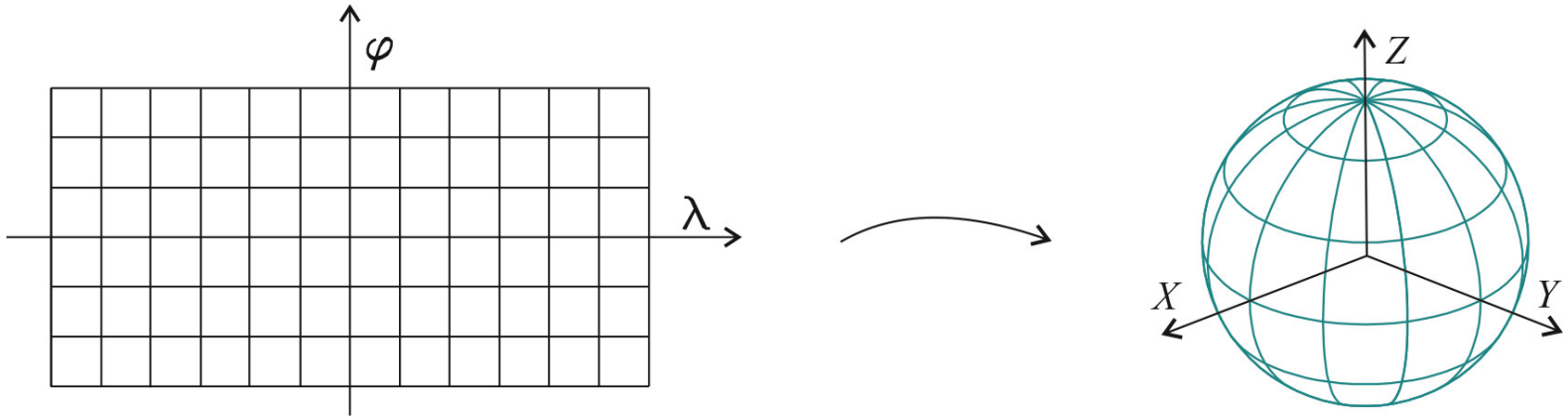
$$X = \cos y \cos x,\ Y = \cos y \sin x,\ Z = \sin y$$

$$x \in [-\pi, \pi],\ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

$$X^2 + Y^2 + Z^2 = 1$$


y
x


φ
λ

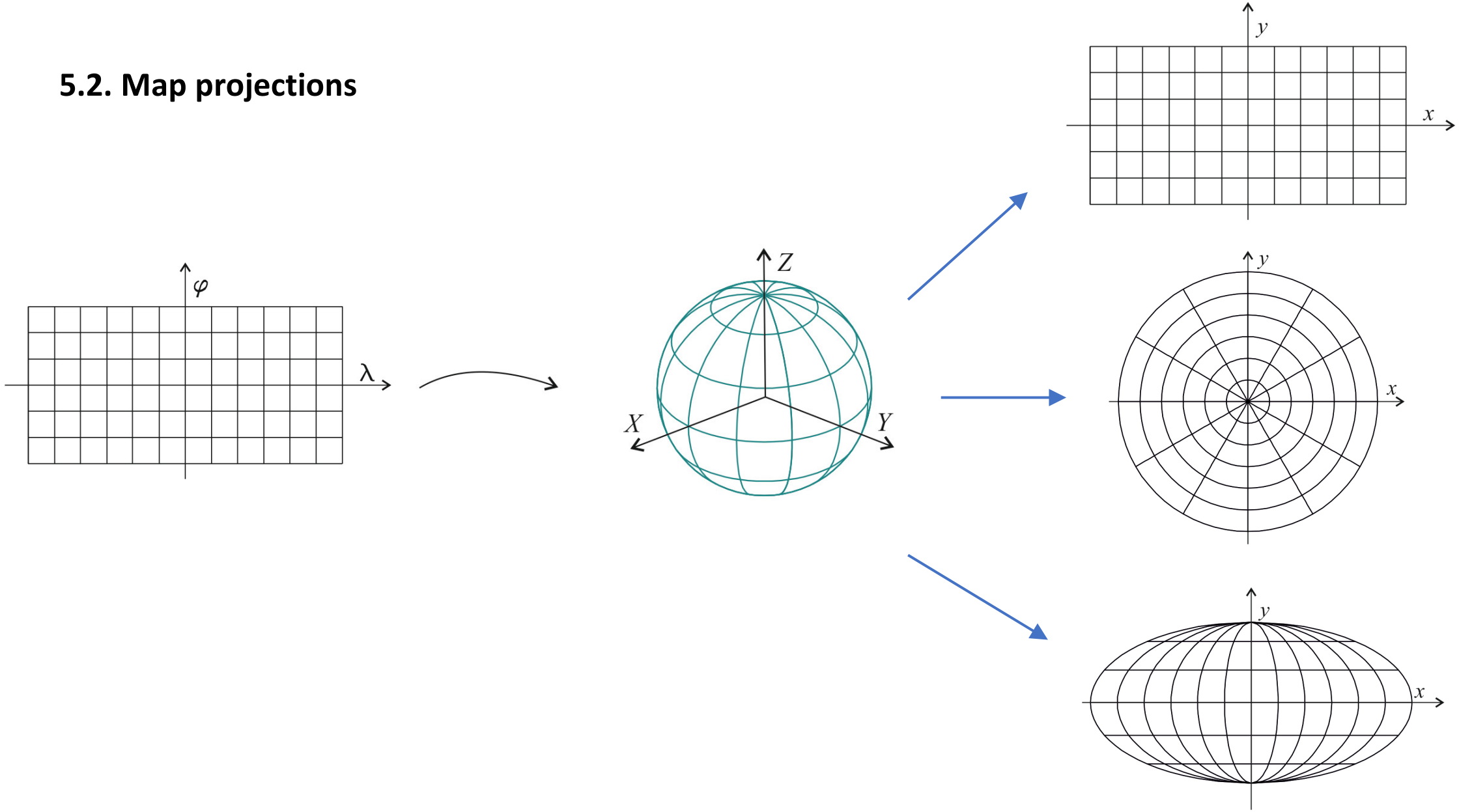


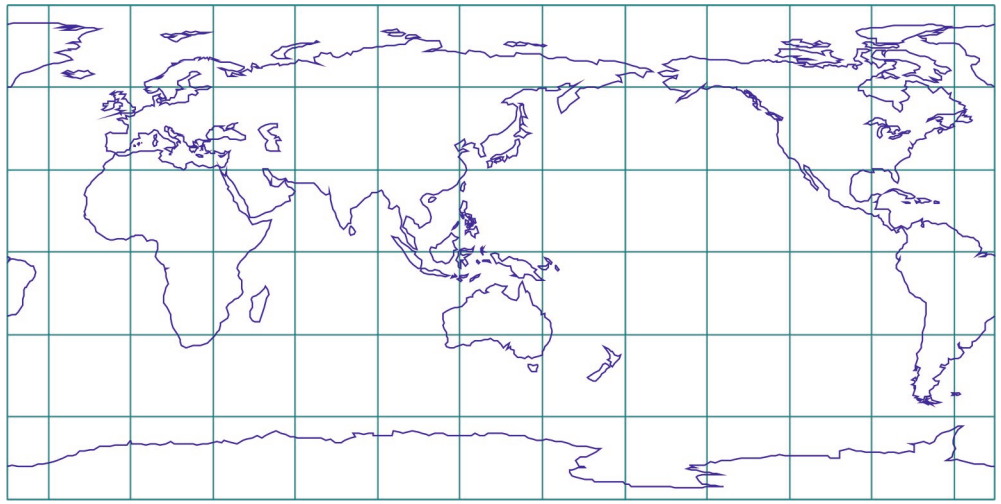
$$X = \cos\varphi\cos\lambda,\ Y = \cos\varphi\sin\lambda,\ Z = \sin\varphi$$

$$X = R\cos\varphi\cos\lambda,\ Y = R\cos\varphi\sin\lambda,\ Z = R\sin\varphi$$

$\lambda \in [-\pi, \pi],\ \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\qquad$ $\varphi \in [-90°, 90°],\ \lambda \in [-180°, 180°]$

5.2. Map projections





$$x = \lambda, y = \varphi$$

World map in a simple cylindrical projection

$$x = \left(\frac{\pi}{2} - \varphi\right)\cos\lambda,\ y = \left(\frac{\pi}{2} - \varphi\right)\sin\lambda$$

The UN flag with the map of the world in an azimuthal projection equidistant along meridians

$$x = \frac{2\sqrt{2}}{\pi}\lambda\cos\beta, \quad y = \sqrt{2}\sin\beta, \quad 2\beta + \sin 2\beta = \pi\sin\varphi$$

The logo of the International Cartographic Association contains a map of the world in Mollweide projection

6. Conclusion

Map projection is the mapping of some curved surface, most commonly a sphere, into a plane

Geographical parameterization of the sphere assums knowledge of secondary school mathematics

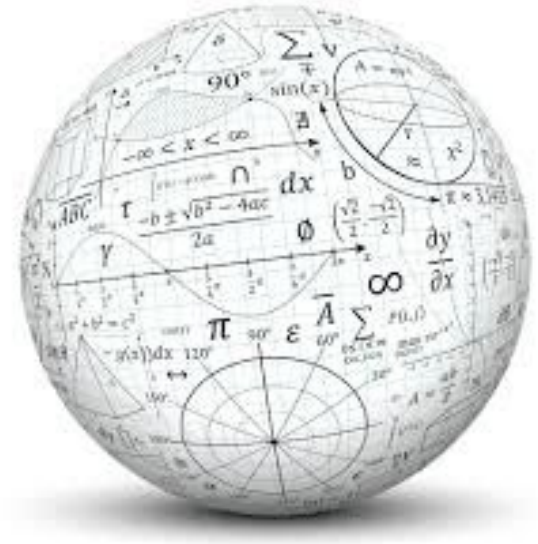
This allows us to define various map projections in a correct way.

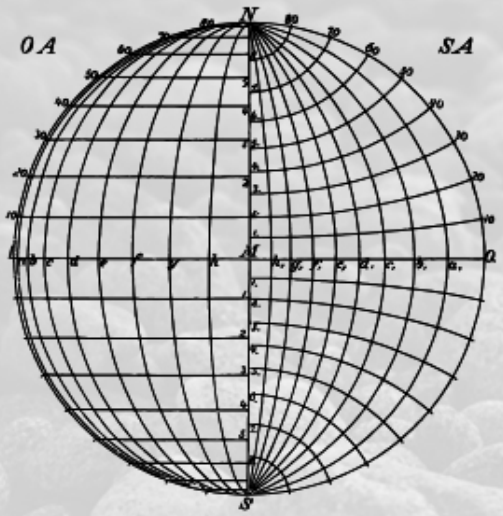
Recommendations

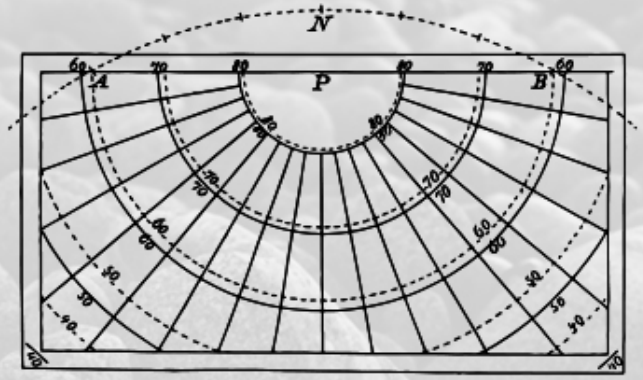
(i) Map projections teaching without mathematics should be avoided

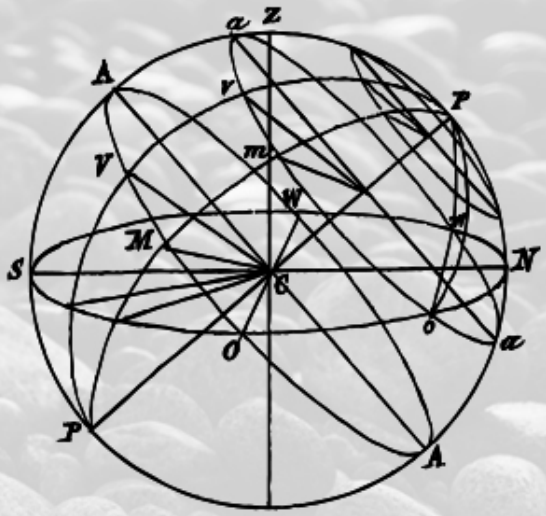
(ii) The word *conceptually* should be left out

(iii) Introducing auxiliary surfaces and secant projections is a big mistake









Thank you!